THE GRAVITY MODEL IN STUDYING THE SPATIAL INTERACTION BETWEEN CITIES: A CASE STUDY OF CONSTANŢA COUNTY, DOBROGEA, ROMANIA

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ABSTRACT: *The gravitational model is the most common tool in urban geography in spatial interaction modeling. This model, derived from Isaac Newton's law of universal gravity, uses two variables to estimate the spatial interaction between at least two points (cities in our case): the number of the population of these cities and the distance between them. The use of gravitational force in predicting spatial interaction dates back to before 1850. This study aims to study the spatial interaction between the cities of Constanţa County, Dobrogea region, Romania through three models proposed by H.C. Carey, W.J. Reilly-P.D. Converse and G.K. Zipf. The spatial interaction between the county-capital city of Constanţa and the other cities in the county weakens with the departure from it, becoming negative. Having the largest number of inhabitants, Constanţa extends its urban influence to the entire county. The only city in the county that comes closest to the zipfian distribution is Mangalia.*

Keywords: *Carey; Constanţa County; gravity model; Reilly-Converse; spatial interaction; Zipf.;*

1. Introduction

The phenomenon of the influence of cities has its roots in antiquity. The Peloponnesian War in ancient Greece, which opposed two of the largest cities of that period, Athens and Sparta, and ended with the victory of Sparta, represented the attempt of the two cities to enlarge, to the detriment of each other, their sphere of commercial and military influence in the Greek world.

One of the problems of urban geography originally derived from physics and which has aroused over time a great interest worldwide among several categories of specialists (economists, geographers, etc.) was the study of the spatial interaction between cities, determining their limit of urban influence.

Over time, various models of interaction between cities have been developed, of delimiting their area of urban influence. Three models were used in this study: Carey, Reilly-Converse, and Zipf, all based on Isaac Newton's law of universal gravity.

The gravity model is one of the basic models of spatial analysis in geography.

2. Materials and methods

2.1. Literature review

The Newtonian gravitational model has been used over several centuries in various sciences. He describes the impact of distance on the interaction between different material bodies (cosmic bodies, groups of people/migration, cities of different sizes, etc.).

The gravitational model was originally introduced into physics in 1686 by Isaac Newton (1643-1727) in his ”Philosophiae Naturalis Principia Mathematica” (Mathematical Principles of Natural Philosophy) by the following formula (Newton, 1686/1999): $F = G*(M1*M2)/r2$ (1), where: F = the power of gravity, M1, M2 = the masses of two interacting bodies, r = the Euclidean space between the two bodies, G = gravitational constant.

According to this formula, a gravitational force acts between two bodies in space that is directly proportional to their mass and inversely proportional to the square of the distance between them.

H.C. Carey (1793-1879), American economist and sociologist, applied for the first time in the social sciences the theory of gravity (Carey, 1858-59).

The German-English geographer E.G. Ravenstein (1834–1913) develops in 1885 the theory of migration in which he describes the spatial interaction between human groups through a gravity model in a study of migration flows between cities in the United Kingdom (Ravenstein, 1885).

Also in the stage of early contributions to the development of the gravity model, one can mention the contribution of the American economist E.C. Young (1892-1968) (Young, 1924), but especially that of the American astrophysicist J.Q. Stewart (1894-1972). Stewart uses concepts and theories of classical physics to explain laws of social physics, for example the demographic behavior of a large group of people in a city that acts as a force of attraction for other groups of people. It elaborates notions and formulas of demographic gravity, demographic force, demographic energy, demographic potential, demographic gradient, etc. (Stewart, 1941, 1948).

Remarkable contributions, also related to Stewart's studies, brought the American economist W.J. Reilly (1899-1970) through the so-called ”the law of retail gravitation” (Reilly, 1931), P.D. Converse, with ”new laws of retail gravitation” (Converse, 1949), and the American linguist G.K. Zipf (1902-1950) (Zipf, 1949).

This topic has been the basis for the development of hundreds of studies and research worldwide in recent decades. In Romania, one can mention the studies and works of I. Iano (1987), who made a map with the theoretical area of influence of the cities of Romania, based on the formula Reilly-Converse, Ionescu-Heroiu, M. et al. (2013) and Cristea, M. et al. (2017), the last two carrying out in three World Bank reports the Zipf distribution of romanian cities in 2010 and 2012, respectively 1992, 2002 and 2011.

2.2. Study area

Dobrogea, known since antiquity as Scithya Minor, is the historical province located in the south-eastern extremity of Romania, between the Danube (to the west), the Black Sea (to the east), Ukraine (to the north) and Bulgaria (to the south) (Fig. 1). It has an area of 15,570 km2, consisting of two counties: Tulcea (in the north) and Constanţa (in the south), with 7,071 km2. The population of the two counties is about 990,000 inhabitants (2021), of which about 760,000 inhabitants in Constanţa County.

In this part of the country, human settlements have developed since the Paleolithic, followed by those of antiquity (Dacian and Roman), the Middle Ages and the modern era. Some cities are located on the hearths of ancient settlements, thus proving a permanent inhabitation of the population in this region.

Currently, in Dobrogea there are 16 cities, of which 11 in Constanţa County, different in number of inhabitants (or demographic size).

Fig. 1. The geographical position of Constanţa County in Europe, Romania and Dobrogea

2.3. Theory and methodology

Specifying the limit of the area of influence between two urban centers becomes difficult from the perspective of the complex relations (of spatial, socio-economic, demographic, administrative, cultural, sanitary, educational nature, etc.) that are established between them and the surrounding area. The area of influence of two nearby urban centers will extend to the limit at which the two cities register the same intensity of relations with the outside or to the limit at which the influence of a city approaches zero (Iano , 1987).

2.3.1. H.C. Carey uses gravitation theory as an analytical concept to interpret the interaction between groups in space in a way analogous to gravity. According to his theory, ”man tends of necessity to gravitate toward his fellow man”, thus forming concentrations of the population (Carey, 1858-59).

It also states that ”gravity is here, as everywhere in the material world, in the direct relation of mass and in the inverse ratio of distance” (Carey, 1858-59).

These ideas can be transposed into a formula that can be used in the spatial interaction between cities, in delimiting their area of urban influence: I = (P1*P2)/d (2), where: I = the spatial interaction (gravity), P1 = the number of the population of the county-capital city of Constanţa (the largest city in the county), P2 = the number of population of a city in the county, d = the linear distance between cities P1 and P2.

2.3.2. W.J. Reilly, American economist, proposed in 1931 the law of retail gravitation, also based on the gravity model of Newton. Reilly attempted to establish an analogy between the commercial attraction of two cities and Newton's law of universal attraction. It was worded in the following way: two cities attract their buyers from the surrounding countryside in direct proportion to their population and inversely proportional to the square of the distance between them. In other words, the influence of a city on the surrounding space decreases with distance and increases by the size of the city.

In 1935, P.D. Converse added a new idea to Reilly's theory: the existence of a ”border point” (or ”zero attraction”) between the two cities, an intermediate point where the number of those who shop in one city is equal to the number of those who shop in the other city. It is thus possible to calculate the distance at which this intermediate point is situated from the two cities and, finally, the area of influence of each of the two cities, according to the formula: dA = dA–B/B[1+(PA/PB)], and dB = dA–B – dA (3), where: dA = distance to city A; dA–B = distance between cities A and B; PA, PB = population of cities A and B.

Based on the above formula, the radius of infl uence was calculated, representing the distance to which the influence of the city is felt. Also, the area of the urban area of influence was calculated starting from the premise that the area of influence has a circular shape (the area of the circle is equal to πr2). The formula takes into account the real distances on the roads.

2.3.3. G.K. Zipf, an American linguist, discovered in 1949, in line with Newton's gravity model, that the frequency of any word is inversely proportional to its rank. In other words, he found that the most common word would appear about twice as often as the second most common word, three times more often than the third most common word, and so on. Zipf's law, formulated in terms of urban geography, indicates an inversely proportional ratio between the rank of a city (r) and its demographic size (x): x = r –1. Zipf's law for a city system can be written like this: Pr = P1/r (4), where: Pr = the population of a city with a certain rank, P1 = the population of the city with the highest rank, r = the rank of a particular city.

Zipf's law detaches the rank-size rule. According to this rule, if the cities of a country are ranked by the number of inhabitants, the population of a city of rank r will be 1/r of the population of the city with the highest rank (”primordial city” – ”primate city”), in the sense of the American geographer Mark Jefferson, meaning ”first” city, of ”1st rank”, which represents the city ”at least twice as large as the next city in size and more than twice as important” (Jefferson, 1939).

In other words, the largest city in a country or region must be twice as large as the second largest city in that country or region, three times larger than the third largest city, and so on.

3. Results and discussions

Table 1 includes the cities in Constanţa County, the number of inhabitants of these cities, the distances (linear and road) between these cities and the county-capital city of Constanţa, as well as the results obtained by applying the formulas (1), (2), and (3) (Anuarul Statistic al României 2019, Google Earth Pro 2022). The number of population of cities is graphically shown in Figure 2.

both those with a larger number of inhabitants (e.g. Năvodari) and those with a smaller number of inhabitants (e.g. Ovidiu) (Matei, 2022). In contrast, these are the small situational towns furthest from Constanţa (Hârşova, Negru Vodă, and even Cernavodă).

From Figure 4 it can be seen that the largest

Table 1. Demographic characteristics of cities in Constanţa County and calculations
(Source: INS 2019)

Crt. no.	Name of the city	Number of inhabit. (a)	A	B	C	D	E	F (b)	G inhab.	G %	H
1	Constanţa (P_1)	312,250	–	–	–	222[1]	79.22	312,250	0	0	1
2	Medgidia	44,722	32	436	2.64	39	13.80	156,125	111,403	71.4	2
3	Năvodari	42,533	19	699	4.23	23	7.96	104,083	61,550	59.1	3
4	Mangalia	40,596	41	309	1.87	43	14.58	78,063	37,467	48.0	4
5	Cernavodă	18,681	52	112	0.67	63	14.96	62,450	43,769	70.1	5
6	Ovidiu	15,890	13	381	2.31	13	2.86	52,042	36,152	69.5	6
7	Murfatlar	11,207	19	184	1.11	17	3.16	44,607	33,400	74.9	7
8	Eforie	11,063	16	216	1.31	16	2.95	39,031	27,968	71.7	8
9	Hârşova	11,045	79	43	0.26	85	15.71	34,694	23,649	68.2	9
10	Techirghiol	8,182	14	182	1.10	15	2.39	31,225	23,043	73.8	10
11	Negru Vodă	5,668	53	33	0.20	57	7.61	28,386	22,718	80.0	11

1 road distance to Bucharest

A – linear distance (km) from the county-capital city (P1); B – spatial interaction in the gravity model: I = (P *P)/d; C – radius (cm) – Carey; D – road distance from Constanţa (km); E – radius of influence (km) – Reilly-Converse; F – the ideal number of inhabitants according to Zipf's law (b); G – difference between (b) and (a); H – rank (r) – Zipf.

From Figure 3 it can be easily seen that the largest spatial interaction (gravity) with the county-capital city of Constanţa is held by the cities located at the shortest distance from it, radius of influence (in km) has, as it was

The city of Constanţa covers the area of the county in which it is located, extending its area of influence even beyond its northern limit, in the southern part of Tulcea County.

On the second place, depending on the radius of influence, are the cities located farther from the county-capital city, regardless of their number of inhabitants. This group includes the cities of Hârşova, Cernavodă, Mangalia, and Medgidia.

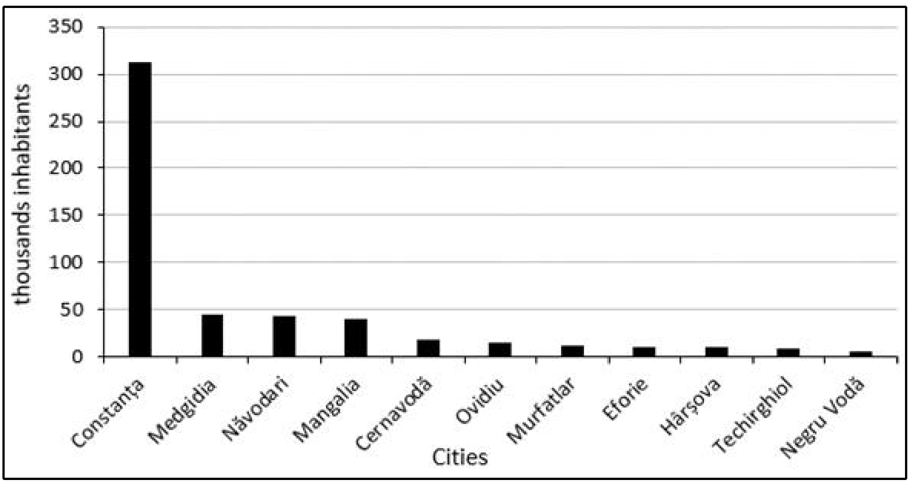

Fig. 2. The number of inhabitants of the cities of Constanţa County
(Source: INS 2019)

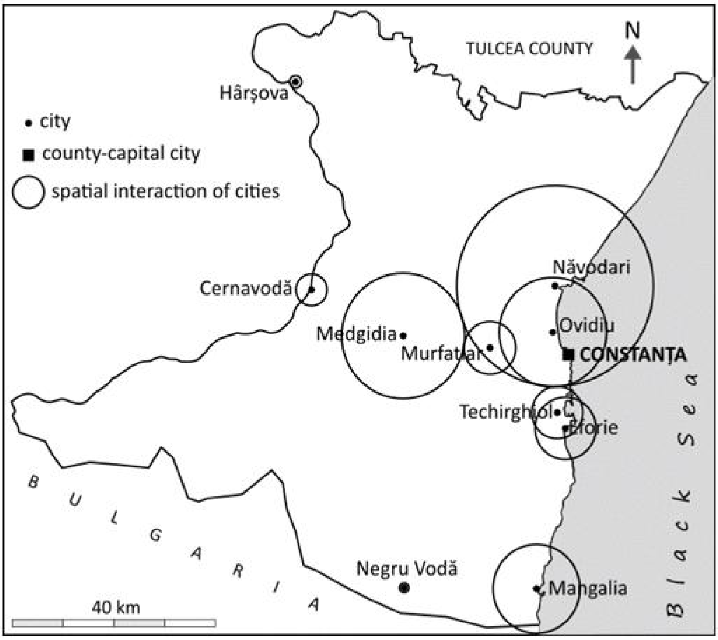

Fig. 3. Spatial interaction of cities in the Carey modelnatural, the county-capital city of Constanţa, which has the largest number of inhabitants in the county (Matei, 2021)

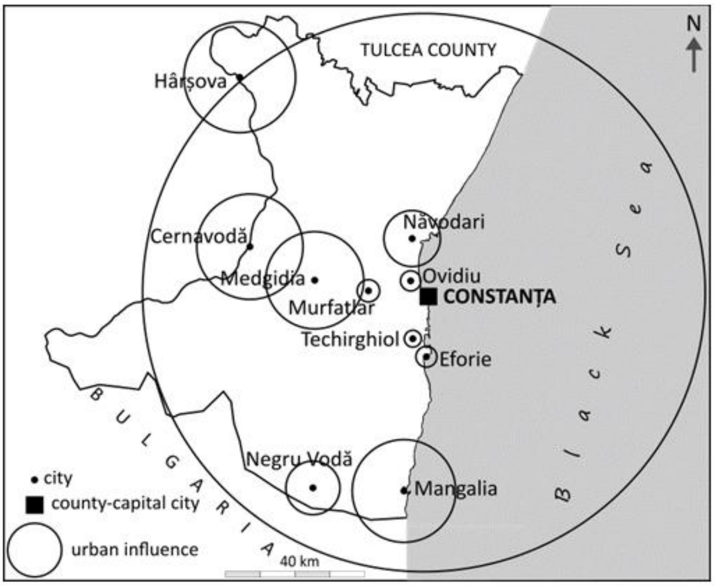

Fig. 4. Urban influence area of cities in the Reilly-Converse model

The third group of cities, with the smallest radius of influence, includes the cities located closer to the county-capital city: Murfatlar, Eforie, Ovidiu, and Techirghiol.

It is also interesting to follow the hierarchy imposed by the area of influence (in km2) of the cities. Thus, only two cities exceed the national average of the surface of the area of influence of the romanian cities (the average theoretical area of the area of influence of the cities in Romania is about 745 km2): Constanţa and Hârşova.

Each city received a rank (r) (Fig. 5) in descending order of the current number of inhabitants (a). For Dobrogea, the city with the 1st rank is Constanţa, the city with the largest number of inhabitants, the primordial city of the region, the city of the 5th rank in the urban system of Romania (2020), where the capital Bucharest, represents the primordial city of the country. The last rank, 11, is assigned to the city of Negru Vodă.

Cities with a smaller number of inhabitants (less than 20,000 inhabitants) have the difference between the population size and the ideal number of inhabitants calculated according to Zipf's higher law (between 68 and 75%).

The city that comes closest to the rank-size rule is Mangalia (with 48%), and the city with the biggest difference is Negru Vodă (with 80%), the city with the lowest number of inhabitants of all the cities of Constanţa County (with only 5,668 inhabitants).

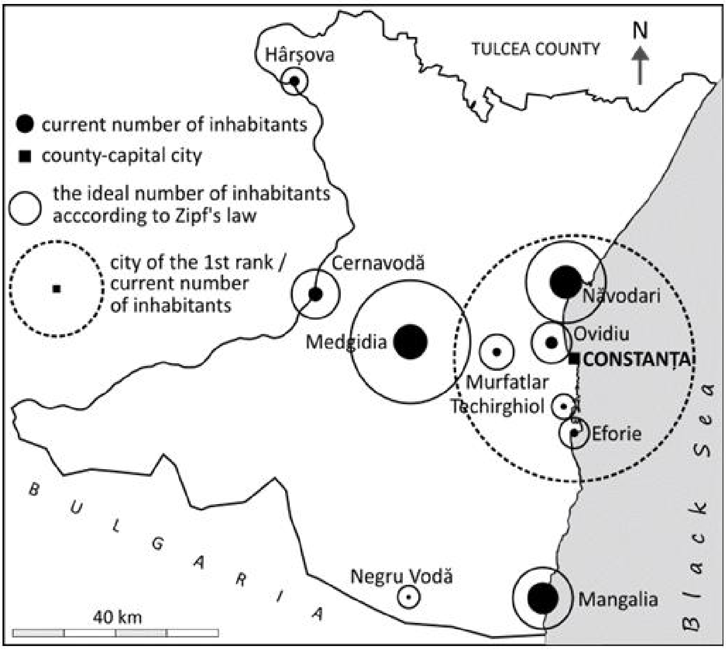

Fig. 5. The demographic rank-size ratio of cities in the Zipf model

The data in Table 1 is graphically transposed into Figure 5, which shows the difference between the current population of cities and their population calculated according to Zipf's law.

From the above representations, an inversely proportional ratio between the rank (size) of the city and the number of population calculated according to Zipf's law is observed.

Thus, cities with a large population of over 40,000 inhabitants, cities of rank 3 and 4 (Năvodari and Mangalia), have the smallest difference (between 48 and 60%) between the current demographic size and the number of inhabitants calculated according to Zipf's law (Matei, 2022). In contrast, the city with the rank 2 in the city system of Constanţa County (Medgidia) has this rather high difference (over 71%).

The rank-size distribution of cities can be demonstrated by graphically representing the natural logarithm (ln) of the rank and size of the city (number of people) in a diagram using the function: ln(demographic size) = $\alpha + \beta$ ln(rank). If the slope of the regression line is equal to – 1, Zipf's law is valid for that country or region.

From Figure 6 it is easily seen that the slope of the regression line has the value of – 1.45 and therefore does not perfectly comply with Zipf's law. Indeed, following this law, the 5th city (Mangalia) should have 62,450 inhabitants, while the current population number of the city is 40,596 inhabitants.

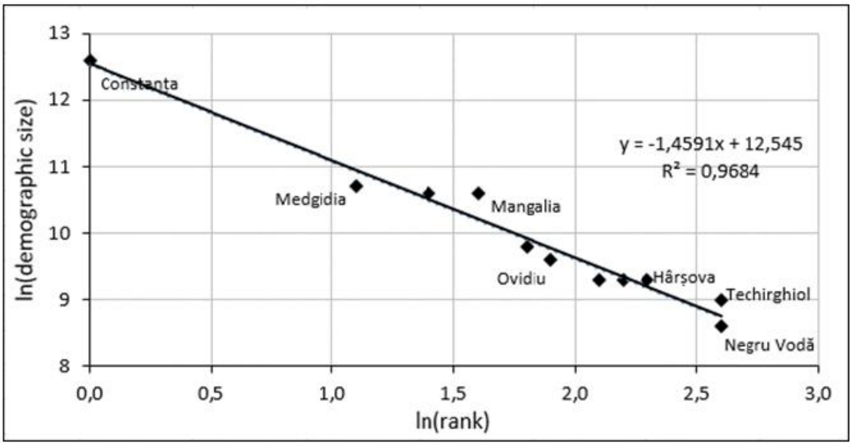

Fig. 6. Rank-size distribution for cities in Dobrogea

On the other hand, in theory, the coefficient of determination R2 has values between 0 and 1. If R2 = 1, the line fits perfectly with the experimental data, and this indicates a linear functional dependence between the variables x and y, that is, each value of variable x corresponds to only one value of variable y. R2 in the case of Dobrogea cities is large (0.968 or almost 97%), which means that the regression line fits the data almost well and explains most of the variation.

Recent research has shown that for most countries of the world the slope is not exactly – 1 and therefore the strict version of Zipf's law is not valid. Zipf's law remains remarkably well globally, but is generally violated at the country level, although it remains valid in certain regions and at certain times (Jiang et al., 2015).

4. Conclusions

The spatial interaction of the cities in Constanţa County depends, according to the Newtonian gravitational model, on the number of population of the constanta county-capital city, on the linear distance between them and the other cities in the county's urban network, but also on the number of the population of these cities.

The cities near the county-capital city of Constanţa (Fig. 3), located on the Black Sea coast, north and south of Constanţa, have a positive spatial interaction with it (Ovidiu). The intensity of the spatial interaction weakens with the distance from the city of Constanţa for the cities located on the Danube-Black Sea channel (e.g. Medgidia), as well as for the cities in the south of the county (e.g. Negru Voda), but especially for the cities located on the Danube (e.g. Hârşova).

With a population of over 300,000 inhabitants, the city of Constanţa overlaps its area of urban influence over the entire area of the county, exceeding even its limits (Fig. 4).
From the perspective of Zipf's model, the urban network of Constanţa County (Fig. 5) does not perfectly follow the zipfian distribution. To fit into this distribution, to achieve their ideal number of inhabitants according to Zipf's law, cities should at least double their number of inhabitants. The closest to this distribution is the city of Mangalia.

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