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ESTABLISHING THE MAXIMUM FLOWS RATES AND VOLUMES FOR FLOODS WITH DIFFERENT PROBABILITIES REQUIRED FOR DIMENSIONING AND EXPLOITATION OF HYDROTEHNICAL CONSTRUCTIONS

Ph.D Eng. CATRINEL RALUCA GIURMA-HANDLEY, Pro. Ph.D Eng. ION GIURMA
"Gheorghe Asachi" Technical University of Iasi, Romania

ABSTRACT: *Knowing the maximum flow rates, as well as the maximum volumes of the floods with different probabilities is necessary in dimensioning and exploitation of hydrotehnical constructions, the safety and efficiency of the works depending greatly on the data accuracy. In the case where measurement data exists in cross-sections location of works, we rely on bidimensional probabilities or correlations between hydrological units. With that in mind, the methodology and a computation example highlighting the two hydrological elements (maximum flow rate and maximum volume) of floods for the desired probabilities are presented in the paper.*

Keywords: *floods; probabilities; dimensioning; hydrotehnical constructions; measurement; bidimensional; hydrological*;

1. Introduction

The floods represents a phenomenon of rapid and significant rise and fall of water flows; they arise from the fall on watershed areas of excessively strong rains, which often overlap a previously recorded soil by rain moistened with a lower intensity. Flows variation during a flood in a section of a river is given by the flow hydrograph called flood hydrograph or flood wave.

Sizing and usage of hydraulic constructions, in addition to knowing the maximum flows in terms of increasing the probability of occurrence, interest and other elements characteristic of floods (shape, volume, total duration, rise time and decreasing time of the flood) (I. Giurma .a., 2008; I, Giurma, 2009).

In determining the appropriate maximum flow of the extremely low probability (1%, 0.5% or 0.01%) can be used in the methods and probabilistics models in the case of existing measurement data or mathematical models and methods in the case of data absence of measurements (R. Giurma Handley .a., 2017).

When it is necessary to know both the maximum flow and volume of the floods with different probabilities in the case of inexisting measurement data, is required the bidimensional probabilities or correlation between the hydrological units (R. Giurma Handley .a., 2017).

2. Bidimensional representations and distributions in hydrology

Bidimensional probability is defined as the probability that the random variables **X** and **Y** take values greater than the reference.

Many hydrological phenomena are, including: maximum flow flood - flood volume flow liquid - solid flow, level - flow etc.

For example it is assumed that a river R. station (section) S, are recorded n floods (n > 20), defined by the flow Q_{max} and volume W_{max}.

Let **A** be an event for which a flood to have a flow rate at the peak (**X**) greater than or equal to Q_{max} $(X \geq Q_{max})$ and **B**, an event for which the same flood to have a volume (**Y**) greater than or equal to W_{max} $(Y \geq W_{max})$.

The problem is to find the flood for the hydrological events **A** and **B**, corresponding to a probability **p=p (A∩B)**, (0,10; 0,05; 0,02; 0,01; 0,005). The calculations are conducted according to the methodology outlined in the specialty literature (S. Hâncu .a., 1971; I. Giurma .a., 2008).

There have been extracted, for a hydrometrical station located on a river, annual maximum flow rates and volumes corresponding to a specific (**n**) period of years (20 years in this example) (Table 1).

$$Q_{med}^{max} = 960/20 = 4 \quad (1)$$

$$W_{med}^{max} = 1707{,}676/20 = 85{,}3828 \; mil.\, m^3$$

The ranges of values $\mathbf{Q_i^{max}}$ and $\mathbf{W_i^{max}}$ had been reverese ordered and modul coefficients $\mathbf{K_{Qi}}$ and $\mathbf{K_{Wi}}$ and empirical probabilities p_i were calculated according to table 1. Were plotted in rectangular axis system the pairs of values ($\mathbf{K_{Qi}}$, $\mathbf{p_i}$) and ($\mathbf{K_{Wi}}$, $\mathbf{p_i}$) (figure 1 and 2) (I. Giurma .a., 2008; R. Giurma Handley, 2017).

Table 1. Elements required for calculation of bidimensional probability

Anul (i)	Q^{max} [m³/s]	W^{max} [mil. m³]	Q_i^{max} ordonat descrescător [m³/s]	W_i^{max} ordonat descrescător [mil. m³]	$K_{Qi}=Q_i^{max}/Q_{med}$	$K_{Wi}=W_i^{max}/W_{med}$	p_i	$u_i=\log(K_{Qi}-C_Q)$	$v_i=\log(K_{Wi}-C_W)$	$u_i\,v_i$
0	1	2	3	4	5	6	7	8	9	10
1	88,4	173,448	182,0	173,448	3,792	2,03	4,762	0,5636	0,4031	0,2271
2	108,0	91,139	108,0	165,249	2,25	1,935	9,524	0,3261	0,3865	0,1260
3	31,7	129,298	106,0	162,726	2,208	1,906	14,286	0,3174	0,3813	0,1280
4	106.0	165,249	90,0	145,381	1,875	1,703	19,048	0,2415	0,3430	0,0828
5	50,6	77,579	88,4	138,758	1,842	1,625	23,810	0,2333	0,3274	0,0764
6	182,0	162,726	50,6	129,298	1,054	1,514	28,571	-0,0348	0,3041	-0,0106
7	9,66	66,856	38,1	97,131	0,794	1,138	33.333	-0,1785	0,2143	-0,0383
8	19,6	97,131	36,4	91,139	0,758	1,067	38,095	-0,2027	0,1951	-0,0395
9	23,2	145,381	34,3	79,471	0.715	0,931	42,857	0,2336	0,1556	-0,0363
10	38,1	79,471	31,7	77,579	0,660	0,909	47,619	-0,2765	0.1489	-0,0412
11	27,3	59,603	28,7	66.856	0,598	0,783	52,381	-0,3307	01082	-0,0358
12	21.4	34,059	27,3	64,649	0,569	0,757	57,143	-0,3585	0,0993	-0,0356
13	17,5	30,275	23,7	59,603	0,494	0,698	61,905	-0,4401	0,0785	-0,0345
14	9,64	16,683	23,2	56,449	0,483	0,661	66,667	-0,4335	0,0648	-0,0284
15	23,7	45,412	21,4	46,673	0,446	0,567	71,429	-0,5017	0,0282	-0,0141
16	36,4	46,673	19,6	45,412	0,408	0,532	76,190	0,5575	0,0137	-0,0076
17	34,3	64,649	17,5	34,059	0,365	0,399	80,952	0,6308	-0,0462	0,0291
18	13,8	26,837	13,8	30,275	0,288	0,355	85,714	-0,8041	-0,068	0,0547
19	90,0	138,758	9,66	26.837	0,2013	0,314	90,476	-1,1530	-0,0894	0,1031
20	28,7	56,449	9,64	16,683	0,2008	0,195	95,238	-1,1561	-0.158	0,1827

The following calculation was carried out for determining the intersection of the maximum flow and maximum annual volumes corresponding to a probability of p = 0.01% (I. Giurma .a., 2008; R. Giurma Handley, 2017).

The calculation begins with determining averages of the two rows of values:

The calculation begins with determining averages of the two rows of values:

From these curves were extracted modul coefficients corresponding to probabilities of 5%, 50% and 95%:

K_{Q5}=3,333 K_{W5}=2,012
K_{Q50}=0,600 KW_{50}=0,845 (2)
K_{Q95}=0,200 K_{W95}=0,220

which were used to calculate the constants $\mathbf{C_Q}$ and $\mathbf{C_W}$, resulting:

$$C_Q = \frac{3{,}333 \cdot 0{,}2 - 0{,}6^2}{3{,}333 + 0{,}2 - 2 \cdot 0{,}6} = 0{,}131 \quad (3)$$

$$C_W = \frac{2{,}012 \cdot 0{,}2 - 0{,}845^2}{2{,}012 + 0{,}2 - 2 \cdot 0{,}845} = -0{,}5$$

Next were determined the following coefficients u_{med}, σ_u, v_{med}, σ_v:

$$u_{med} = \log(0{,}6 - 0{,}131) = -0{,}32 \quad (4)$$

$$\sigma_u = 0{,}304 \log \frac{3{,}333 - 0{,}131}{0{,}2 - 0{,}131} = 0{,}507$$

$$v_{med} = \log(0{,}845 - 0{,}5) = 0{,}129$$

$$\sigma_V = 0{,}304 \log \frac{2{,}012 - 0{,}5}{0{,}2 - 0{,}5} = 0{,}165$$

The arrays of values $\mathbf{u_i}$, $\mathbf{v_i}$ and $\mathbf{u_i v_i}$ were calculated (table 1). Among the variables $\mathbf{u_i}$ şi $\mathbf{v_i}$ correlation coefficient was established:

$$\rho = \frac{0{,}68 - 20 \cdot (-0{,}329)0{,}129}{20 \cdot 0{,}507 \cdot 0{,}165} = 0{,}9138 \quad (5)$$

For the probability p = 0,01 and the correlation coefficient ρ= 0,9138 using the curve of figure 3, results the curve of intersection of the plane [ξ,η], given by the following pairs of values:

ξ=1,7 η=2,27

ξ=1,8 η=2,23

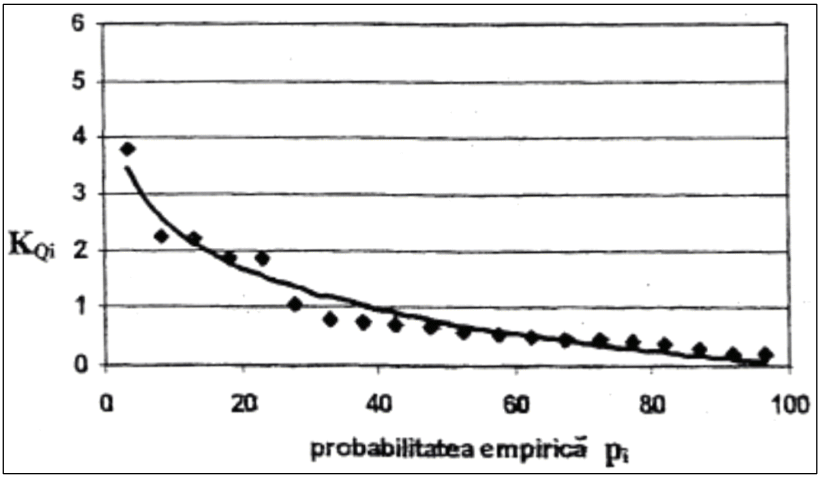

Fig. 1. Empirical probability curve of modul coefficients K_{Qi}

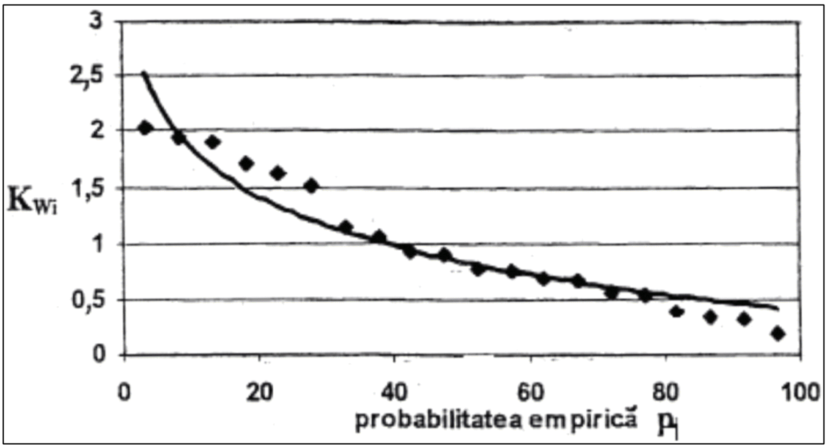

Fig. 2. Empirical probability curve of modul coefficients K_{Wi}

$\xi=1{,}9$ $\eta=2{,}2$
$\xi=2{,}0$ $\eta=2{,}15$ (6)
$\xi=2{,}1$ $\eta=2{,}05$
$\xi=2{,}2$ $\eta=1{,}85$

$Q_{max}=48 \cdot 3{,}9645=190{,}296$ m^3/s
$W_{max}=85{,}3838 \cdot 2{,}6398=225{,}3962$ mil m^3

(3) $u=-0{,}329+0{,}507 \cdot 1{,}9=0{,}6343$

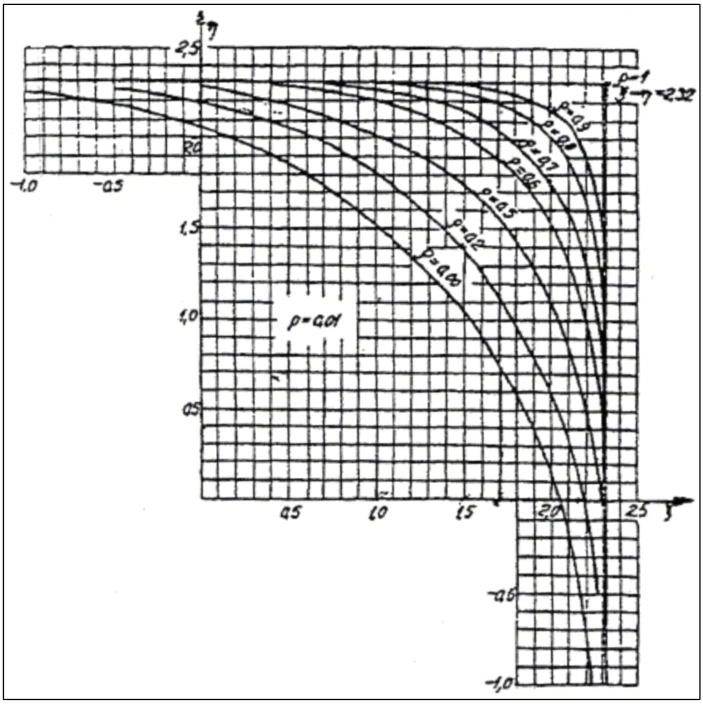

Fig. 3. The curve of intersection [ξ,η] for p=0,01

With the curve of intersection of the plane [ξ,η], were obtained the curve of intersection in the plane [**u,v**], the curve of intersection in the plane [$\mathbf{K_{Qmax}}$, $\mathbf{K_{Wmax}}$] and the curve of intersection in the plane **[$\mathbf{Q_{max}}$, $\mathbf{W_{max}}$]** based on the following calculations (I. Giurma .a., 2008; R. Giurma Handley, 2017):

(1) $u = -0{,}329+0{,}507 \cdot 1{,}7=0{,}5329$
$v=0{,}129+0{,}165 \cdot 2{,}27=0{,}5036$
$K_{Qmax}=0{,}131+ 10^{0{,}5329} =3{,}5421$
$K_{wmax}= -0{,}5+ 10^{0{,}5036} =2{,}6886$
$Q_{max}=48 \cdot 3{,}5421=170{,}02$ m^3/s
$W_{max}=85{,}3838 \cdot 2{,}6886=229{,}5629$ mil m^3

(2) $u=-0{,}329+0{,}507 \cdot 1{,}8=0{,}5836$
$v=0{,}129+0{,}165 \cdot 2{,}23=0{,}4969$
$K_{Qmax}=0{,}131+ 10^{0{,}5836} =3{,}9645$
$K_{wmax}= -0{,}5+ 10^{0{,}4969} =2{,}6398$ m^3/s

$v=0{,}129+0{,}165 \cdot 2{,}2=0{,}492$
$K_{Qmax}=0{,}131+ 10^{0{,}6343} =4{,}4392$
$K_{Wmax}=-0{,}5+ 10^{0{,}492} =2{,}6046$
$Q_{max}=48 \cdot 4{,}4392=213{,}08$ m^3/s
$W_{max}=85{,}3838 \cdot 2{,}6046=222{,}3906$ mil m^3

(4) $u=-0{,}329+0{,}507 \cdot 2=0{,}685$
$v=0{,}129+0{,}165 \cdot 2{,}15=0{,}4838$ (7)
$K_{Qmax}=0{,}131+ 10^{0{,}685} =4{,}9727$
$K_{Wmax}=-0{,}5+ 10^{0{,}4838} =2{,}5465$
$Q_{max}=48 \cdot 4{,}9727=238{,}6892$ m^3/s
$W_{max}=85{,}3838 \cdot 2{,}5465=217{,}4298$ mil m^3

(5) $u=-0{,}329+0{,}507 \cdot 2{,}1=0{,}5329$
$v=0{,}129+0{,}165 \cdot 2{,}05=0{,}5036$
$K_{Qmax}=0{,}131+ 10^{0{,}5329}=3{,}5723$
$K_{Wmax}=-0{,}5+ 10^{0{,}5036} =2{,}4329$
$Q_{max}=48 \cdot 3{,}5723=267{,}47$ m^3/s
$W_{max}=85{,}3838 \cdot 2{,}4329=207{,}73$ mil m^3

(6) $u=-0,329+0,507 \cdot 2,2=0,7864$

$v=0,129+0,165 \cdot 1,85=0,4343$

$K_{Qmax}=0,131+10^{0,7864}=6,246$

$K_{Wmax}=-0,5+10^{0,4343}=2,2183$

$Q_{max}=48 \cdot 6,246=299,8\ m^3/s$

$W_{max}=85,3838 \cdot 2,2183=189,407$ mil m^3

Representing the coordinate points **(W_{max}, Q_{max})** in a rectangular coordinate system, is obtained the curve of intersection **$Q_{max} = f(W_{max})$** for the probability p=0,01 (figure 4) (I. Giurma .a., 2008; R. Giurma Handley, 2017).

The points of intersection of this curve with the coordinate axes define the values of the elements of the flood, which for example account are considered:

$$\mathbf{Q_{max\ 1\%} = 300\ m^3/s} \quad (8)$$
$$\mathbf{W_{max\ 1\%} = 230\ mil\ m^3}$$

For the correlation to be acceptable, the value of the r coefficient should be between 0,7 and 1,0.

For the flow and volume values in table 1 are calculated the units shown in table 2 (I. Giurma .a., 2008).

The correlation coefficient is:

$$r=\frac{20\cdot 111800,8064-960\cdot 1707,676}{\sqrt{(20\cdot 83650,84-960^2)\cdot(20\cdot 193757,5-1707,676^2)}}$$

$$r=\frac{2236016,128-1639368,96}{\sqrt{(1673016,8-921600)\cdot(3875150-2916157,320976)}}$$

$$r=\frac{596647,168}{848883,5039715}=0,702 \quad (10)$$

The correlation coefficient is at the lower limit of what looks like it can provide a relatively good correlation between the two hydrological units.

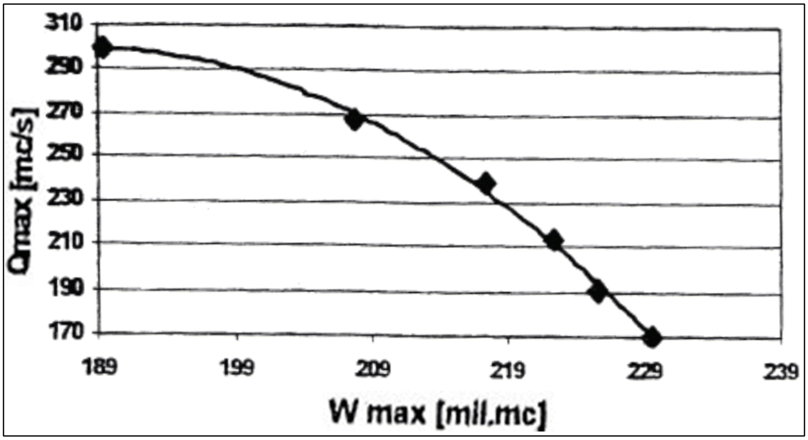

Fig. 4. The curve of intersection Qmax =f(Wmax) for the probability p=0,01

3. Simple linear correlation of two hydrological units

Linear relationship between two random variables can be appreciated by means of a correlation coefficient, which in the case of flood flow and volume, has the following form (I. Giurma, R. Drobot, 1987; I. Vladimirescu, 1984):

$$r=\frac{n\sum_{i=1}^{n}Q_iW_i-\sum_{i=1}^{n}Q_i\cdot\sum_{i=1}^{n}W_i}{\sqrt{\left[n\sum_{i=1}^{n}Q_i^2-(\sum_{i=1}^{n}Q_i)^2\right]\cdot\left[n\sum_{i=1}^{n}W_i^2-(\sum_{i=1}^{n}W_i)^2\right]}}$$

For a linear correlation, the bond between x and y is (C. Diaconu, 1999; A. Stoianovici .a., 1998):

$$\mathbf{y=a+b\ x} \quad (11)$$

For a particular value **x=xi** we note the measured value of the variable analyzed $\mathbf{y_i^m}$ and $y_i^c = a + b\,x_i$ theoretical value of the variable y, wich verify the equation of the regression line.

Table 2. The necessary elements for the calculation of the correlation coefficient r

Anul (i)	Q_i [m^3/s]	W_i [mil. m^3]	Q_i^2	W_i^2	$Q_i W_i$
0	1	2	3	4	5
1	88,4	173,448	7814,56	30084,21	15332,8
2	108,0	91,139	11664	8306,317	9843,012
3	31,7	129,298	1004,89	16717,97	4098,747
4	106,0	165,249	11236	27307,23	17516,39
5	50,6	77,579	2560,36	6018,501	3925,497
6	182,0	162,726	33124	26479,75	29616,13
7	9,66	66,856	93,3156	4469,725	645,829
8	19,6	97,131	384,16	9434,431	1903,768
9	23,2	145,381	538,24	21135,64	3372,839
10	38,1	79,471	1451,61	6315,64	3027,845
11	27,3	59,603	745,29	3552,518	1627,162
12	21,4	34,059	457,96	1160,015	728,8626
13	17,5	30,275	306,25	916,5756	529,8125
14	9,64	16,683	92,9296	278,3225	160,8241
15	23,7	45,412	561,69	2062,25	1076,264
16	36,4	46,673	1324,96	2178,369	1698,897
17	34,3	64,649	1176,49	4179,493	2217,461
18	13,8	26,837	190,44	720,2246	370,3506
19	90,0	138,758	8100	19253,78	12488,22
20	28,7	56,449	823,69	3186,49	1620,086
	$\sum Q_i = 960$	$\sum W_i = 1707{,}676$	$\sum Q_i^2 = 83650{,}8$	$\sum W_i^2 = 193757{,}5$	$\sum Q_i W_i = 111800{,}8064$

The parameters a and b can be determined by the method of least squares sothat the sum of the squares of the deviations of of the $\mathbf{y_i^c}$ and $\mathbf{y_i^m}$ values to be minimal:

$$S = \sum_{i=1}^{n} (y_i^c - y_i^m)^2 = \sum_{i=1}^{n} (a + bx_i - y_i^m)^2 = min \qquad (12)$$

Stationary points of the function are obtained by canceling its partial derivatives in conection with the unknowns values, that is to say in connection to **a** and **b**:

$$\begin{cases} \frac{\partial S}{\partial a} = 2\sum_{i=1}^{n}(a + bx_i - y_i^m)\cdot 1 = 0 \\ \frac{\partial S}{\partial b} = 2\sum_{i=1}^{n}(a + bx_i - y_i^m)\cdot x_i = 0 \end{cases} \qquad (13)$$

It can be rewritten by arranging terms:

$$\begin{cases} na + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i^m \\ a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i^m \end{cases} \qquad (14)$$

If we consider $\mathbf{x_i{=}Q_i}$ şi $\mathbf{y_i{=}W_i}$, we obtain the following system of equations:

$$\begin{cases} 20\,a + 960\,b = 1707{,}676 \\ 960\,a + 83650{,}84\,b = 111800{,}8064 \end{cases} \qquad (15)$$

$$a = \frac{\begin{vmatrix} 1707{,}6766 & 960 \\ 111800{,}8064 & 83650{,}84 \end{vmatrix}}{\begin{vmatrix} 20 & 960 \\ 960 & 83650{,}84 \end{vmatrix}} = \qquad (16)$$

$$= \frac{1707{,}676\cdot 83650{,}84 - 960\cdot 111800{,}8064}{20\cdot 83650{,}84 - 960^2} = 47{,}27$$

$$b = \frac{\begin{vmatrix} 20 & 1707{,}676 \\ 960 & 111800{,}8064 \end{vmatrix}}{\begin{vmatrix} 20 & 960 \\ 960 & 83650{,}84 \end{vmatrix}} = \qquad (17)$$

$$= \frac{20\cdot 111800{,}8064 - 960\cdot 1707{,}676}{20\cdot 83650{,}84 - 960^2} = 47{,}27$$

The regression equation will be:

$$W = 47{,}27 + 0{,}794\,Q \qquad (18)$$

To establish the line, we use two pairs of the representative values (Q_1=9,66; W_1=54,94) and (Q_2=20; W_2=63,15) (fig, 5).

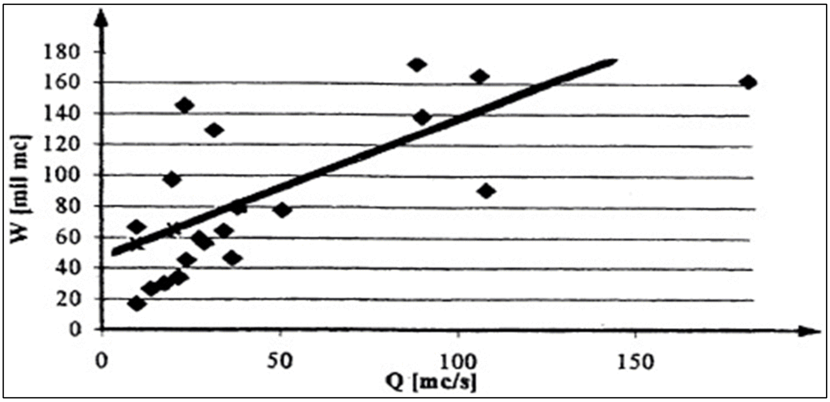

Fig. 5. Regression line W=W(Q)

It can be withdrawn and the line **Q=Q(W)** following the same steps considering the equation xi=Wi and yi=Qi.

4. Conclusions

a) Maximum flows and volumes with different probabilities are therefore the most important elements of flood waves used in dimensioning and usage of hydrotehnical constructions; safety and economic efficiency of them depends largely on the accuracy of these values.

b) In case of the existing measurement data, to obtain values closer to reality, we recommend using the bidimensional representations and distributions or correlations.

c) Dimensional probability of the hydrological presents a particular interest in practice because many hydrological phenomena are of bidimensional (for example: maximum flow of a flood and volume of flood wave); it is presented an example of this methodology and the resulting calculation of the curve of intersection of the two hydrological units with the probability p = 0,01, the values obtained having a high precision.

d) When the number of measurements is reduced, the idea of defining a bidimensional distribution densities is off, being preferred to rely on correlations and is presented a calculation example and the methodology.

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